MATHEMATICAL MODEL OF INTERACTION OF SOLID BODIES WITH SOIL UPON THEIR RELATIVE DISPLACEMENT

K. S. Sultanov

UDC 622.011.4+622.023

The problem on interaction of seismic and blast waves with various underground constructions in the soil media are usually solved under the assumption that the relative displacement or shear stress (referred to as classical) is equal to zero at the interface between a construction and the soil, which is sometimes violated in actual practice. The results of analysis of earthquakes aftereffects [1] and experimental investigations [2] demonstrate that the interaction of seismic and blast waves with underground construction generates considerable relative displacements up to the rupture of links between the soil and the outer surface of an underground construction. These results necessitate the investigation of regularities in the shear interaction of solids with the soil and the development of conditions alternative to classical conditions at the boundaries of the contact surface.

Experimental investigations of the interaction between foundations and ground during the relative displacement were carried out in [3], where a Winkler-type condition was proposed for describing the interaction process on the basis of experiments. The application of this condition is demonstrated for the problems on shear interaction of underground constructions with soil under seismic loading [4]. The experiments [5] aimed at the investigation of shear interaction of underground constructions with soil show that a Winkler-type condition describes only the initial stage of the shear interaction for small values of strain. In [2, 4, 5], the experiments were carried out with thin tubes, while shear interaction of large-scale underground construction with the soil had not been investigated experimentally. Besides, a mathematical model describing the regularities of shear interaction correctly has not been developed so far.

The present paper is devoted to experimental investigation of shear interaction of underground construction with the soil and to the derivation of analytical relations describing these regularities.

1. EXPERIMENTAL TECHNIQUE AND RESULTS

Experimental investigations of shear interaction of underground constructions with the soil [2, 4, 5] indicate that the derivation of regularities governing the variation of shear stresses at the contact surface between a construction and the soil under natural conditions is a complicated and labor-consuming problem. In the case of large-scale underground constructions, these difficulties are multiplied. However, the experimentally obtained [2, 4, 5] dependences of shear stresses on relative displacement (shear) at the contact surface between the soil and the construction revealed that they do not differ qualitatively from similar dependences obtained during the shearing of soil and rock samples [6-10] on shear test instruments of various constructions and operation principles [6, 8]. Following these works as well as [11], we determined experimentally the regularities of interaction between elements of large scale underground constructions, which will henceforth be referred to as solids, and the soil by using a plane-shearing instrument VSV-25. The description of this instrument is given in [8]. The application of shearing instruments in the experiments aimed at determining the regularities of interaction between solids and the soil makes it possible to carry out a series of experiments under laboratory conditions (each experiment with identical initial parameters was repeated 10-15 times). This simplifies considerably the experimental setup and improves the accuracy of the obtained results.

We carried out the experiments with preliminarily prepared samples of solids in the form of cylindrical disks with diameter $D = 0.07$ m, height $h = 17\cdot 10^{-3}$ m, and area of contact with soil $S = 0.004$ m^2, which densely fill the cylinder of the soil-inlet chamber of the

Tashkent. Translated from Prikladnaya Mekhanika i Tekhnicheskaya Fizika, No. 1, pp. 40-48, January-February, 1993. Original article submitted October 4, 1991.

shearing instrument. The upper fixed cylinder of the inlet chamber was filled with the soil. The experiments were carried out in the cases when the structural bond between the soil and the solid is destroyed and when it is not destroyed. In the experiments only soil deformation was observed, while the solids were not deformed or destroyed.

In the first case, we used the soil with a distorted structure, and samples of solid and soil were loaded consecutively in the soil-inlet chamber, while in the second case the adhesion between the samples of solid and soil was created artificially by preliminary wetting with water till saturation and subsequent drying. Then the solid sample with undistorted structural bonds and the soil which also had an undistorted structure were placed together in the soil-inlet chamber of the shear instrument. In both cases, the height of the soil sample was 0.035 m.

Undulating roughness is created preliminarily on the surface of the solid sample which is in contact with the soil. After the instrument is assembled completely, the lower cylinder of the instrument is shifted relative to the upper cylinder, and the solid body is shifted relative to the soil. The shearing force is produced by an electric motor through a special attachment. The shear rate remains unchanged during one experiment. The experiments were carried out for two values of shear rate: $v = 3 \cdot 10^{-4}$ m/sec (quasistatic mode, duration of experiment 40 sec), and $v = 3 \cdot 10^{-2}$ m/sec (dynamic mode, duration of experiment 0.4 sec). The rate was changed in various experiments by replacing the attachments connecting the motor with the shearing instrument.

The values of shearing force, the pressure normal to the contact surface between the soil and the solid, and the displacement of the lower cylinder relative to the upper one were measured in experiments with the help of special pickups. The normal pressure was measured both over the sample, where it is applied, and directly at the surface of contact between the soil and the sample. The normal pressure over the soil sample and the shearing force were measured by annular strain gauges, while the measurements at the contact between the soil and the solid were carried out with a membrane-type gauge indented in the solid sample. Relative displacement was measured by a slide-wire gauge. The signals from the gauges were recorded on a photographic paper by a mirror-galvanometer oscillograph N-117. This method of measurements of the values of parameters differing from traditional methods [8] improves the accuracy of the quantities being measured and rules out the errors that can hardly be avoided in visual fixation of the values of parameters, especially during high-rate shearing. The oscillograms obtained in this way make it possible to construct the dependences of shear stresses τ at the surface of contact between the soil and the solid on the strain u. The experiments were carried out with loess, loam, and sand with specific gravity γ = 1500-1600; 1600-1700, and 1650-1750 kg/m^3 having a humidity W = 8-10%. These characteristics were measured after the experiments in the case of an undistorted bond between the soil and the solid (interaction of a solid with the soil having an undistorted structure) and before the experiment in the case of a distorted bond (interaction of a solid with the soil of a distorted structure). The values of normal pressure, shear rate, and the roughness of the contact surface between the solid and the soil varied from experiment to experiment. The experiments were carried out mainly for two kinds of surfaces: rough surfaces, when the height of undulating periodic protrusions was δ = 0.002 m, and smooth surfaces with δ = 0.0002 m. Henceforth, we shall refer to these samples as rough and smooth solids. In experiments on interaction between solids and soils with a distorted structure in quasistatic mode, the normal pressure was maintained constant, while in other cases the value of normal pressure varied during the shear of a solid relative to the soil.

A typical dependence of the shear stress τ on the strain u obtained during quasistatic interaction between a smooth solid body and a loess soil with a distorted structure is presented in Fig. 1 (curve 1). The $\tau(u)$ dependence has two stages of interaction: (1) the value of τ increases with u, and (2) after attaining a certain value of strain u_*, the shear stress attains a limiting value which remains unchanged upon a further increase in u (second stage). The normal pressure over the soil sample was maintained at about σ_N = 0.15 MPa (curve 2). The normal pressure over the soil sample drops (curve 3). With increasing roughness coefficient of the surface of contact between the solid and the soil, the behavior of curve 3 changes. During interaction of rough solids with the soil, the normal pressure over the soil sample increases at the beginning of the process, attains its maximum value exceeding the normal pressure over the sample, and then drops, i.e., has a dome-shaped dependence. Such a variation in the normal pressure over the soil sample can be explained by dilatational

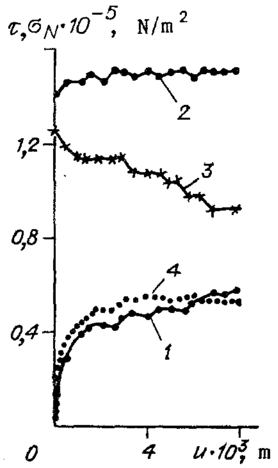

Fig. 1

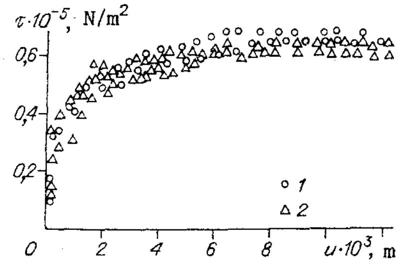

Fig. 2

and deformational characteristics of the soil samples. The behavior of the $\tau(u)$ curve remains unchanged. It is possible to maintain a constant normal pressure over a soil sample under the quasistatic mode of interaction between solids and soils, but under dynamic conditions this cannot be done in some cases. According to [12], a transition to the reduced dependences $\tau^0(u) = \tau(u)/\sigma_N(u)$ (curve 4) practically does not change the behavior of the $\tau(u)$ curve. The function $\sigma_N(u)$ corresponds to the variation of the normal pressure over the soil sample (curve 3). The $\tau(u)$ and $\tau^0(u)$ dependences obtained during interaction of solids with loess soils as well as sand and loam with a distorted structure for various values of the normal pressure over the soil sample, the roughness coefficient of the surface of the solid, and the rate of interaction are qualitatively similar to corresponding dependences presented in Fig. 1, but differ quantitatively.

Figure 2 shows the $\tau(u)$ dependences obtained during the interaction of a rough solid with a loess soil having a distorted structure under a normal pressure $\sigma_N = 0.075$ MPa in the quasistatic and dynamic (points 1 and 2) cases. It can be seen that a change in the interaction rate by two orders of magnitude does not affect the $\tau(u)$ dependence for a soil with a distorted structure.

The $\tau(u)$ dependences obtained during the interaction of a solid with a soil having a distorted structure for $\sigma_N = 0.15$ MPa under dynamic and quasistatic conditions of interaction are presented in Figs. 3 and 4, respectively. Curves 1 and 2 in Fig. 3 correspond to the results of experiments with smooth bodies, and 3 and 6 with rough bodies, while in Fig. 4 curves 1-3 correspond to smooth and 4, 5 to rough solid bodies.

The $\tau(u)$ dependences presented in Figs. 3 and 4 are characterized by the emergence of a peak value τ_p of the shear stress, which is also observed upon a shift of the samples of soils and rocks themselves [6-10]. In both cases, the emergence of τ_p is associated with the accumulation of plastic deformations and with the violation of the structure of the soil. As the strain attains the value u_*, the structural bonds between the soil and a solid are distorted completely, and the shear stress value $\tau = \tau_r$ remains unchanged upon a further increase in the displacement of the solid relative to the soil. According to the results of experiments presented in Figs. 3 and 4, the value of u_* does not change for different rates of interaction. The constancy of u_* for rock samples with different shear rates was also noted in [13]. An increase in the roughness coefficient of the solid surface leads to an increase in the values of τ_p and τ_r (see Figs. 3 and 4). In the limiting case when the roughness coefficient of the solid surface is very large, the values of τ_p and τ_r become equal to those obtained by shifting the samples of soil with an undistorted structure themselves. During repeated interactions of solids with the soils with a distorted structure, the value of τ_r remains unchanged, and the peak strength is not observed. In such cases, the $\tau(u)$ dependences are similar to those presented in Figs. 1 and 2. A change in the normal pressure σ_N during the interaction of solids with soils having an undistorted structure does not affect the behavior of $\tau(u)$ either. The decrease in the value of τ_r observed in experiments under dynamic conditions of interaction is associated with a decrease in σ_N (see Fig. 3). The spread in the results of experiments with identical initial parameters in Figs. 3

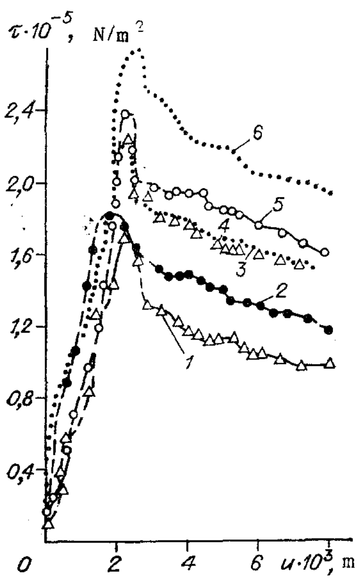

Fig. 3

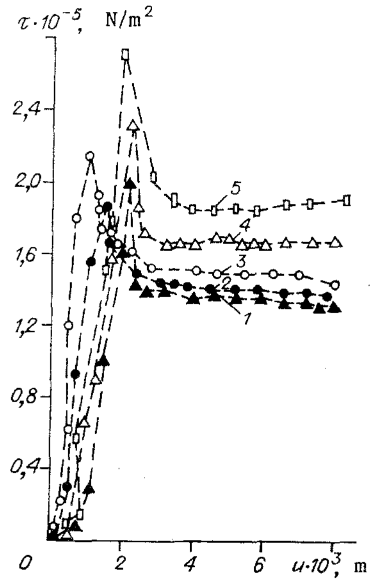

Fig. 4

and 4 can be explained by the variability of artificially created structural bonds between the soil and the solid.

2. MATHEMATICAL MODEL OF INTERACTION

The construction of a mathematical model of deformation of soils under shearing stresses taking dilatational properties of the medium into account was considered in [14] also containing a review of publications devoted to this problem. A new law of friction during the interaction of high- and low-strength rocks and soils for limiting shear stresses in the case of rock falls and landslides was derived in [15]. Tseitlin and Kasheleva [16] proposed a model of interaction of a pile with the soil taking into account some experimentally observed factors including the emergence of the peak strength. A dislocation model of metal fracture proposed in [17] also takes into account the emergence of the peak strength of the material. The regularities of deformation of soils under different types of loading including shearing forces are considered in [18]. The models analyzed in [16] and [17] take into account some of the experimentally observed peculiarities of the interaction between solids and soils. However, these models are either very cumbersome, or involve physical parameters which are difficult to determine during the interaction of solids with the soil.

An analysis of the results of experiments presented in Figs. 1-4 shows that the behavior of the dependences $\tau(u)$ is mainly determined by the extent of distortion of the soil structure, the value of the normal pressure at the soil—solid contact surface, the roughness coefficient of the solid, and the rate of relative displacement (shear). The main factor in the interaction of solids with the soil having an undistorted structure is the emergence of the peak strength of the soil (see Figs. 3 and 4). This property is not observed during the interaction of solids with soils having a distorted structure (see Figs. 1 and 2). It follows hence that the emergence of a peak strength of soils is a result of a change in the structure of the soils in the contact layer during the interaction of solids with the soils having an undistorted structure. For small values of relative displacement, the soil is deformed elastically, and then ductility is manifested in the course of accumulation of plastic deformations. After the attainment of the peak value τ_p of the shear stress, the fracture of the soil in the contact layer is observed. When the relative displacement attains the value equal to u_*, the structure of the soils in the contact layer is violated completely. Consequently, the values of mechanical parameters of the soil (shear modulus, viscosity, etc.) vary during the interaction, i.e., are the functions of the change in the structure of the soil.

The variation of the shear modulus $K_x{}^s$ of interaction obtained from the experimental dependences $\tau(u)$ ($K_x{}^s = \tau/u$) is presented in Fig. 5. Here, curve I corresponds to the interaction of solids with a soil having an undistorted structure, while curves II and III correspond to a distorted structure. Points 1 and 2 correspond to the results of experiments ob-

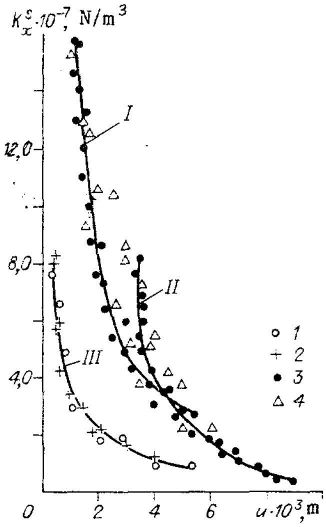

Fig. 5

tained at $\sigma_N = 0.075$ MPa, and points 3 and 4, at $\sigma_N = 0.15$ MPa. It can be seen that the variation of $K_x{}^s$ with the structure of the soil is of nonlinear type. The range of variation of $K_x{}^s$ for an undistorted structure of the soil is wider than for those with a distorted structure. The variation of σ_N does not affect the behavior of $K_x{}^s(u)$ curves.

According to the results of experiments, the motion of a solid relative to the soil with a completely distorted structure ceases immediately after unloading, and the value of shear stress also drops to zero. For a soil with an undistorted structure, a return motion is observed. The value of the reverse displacement u_R depends on the extent of distortion of the soil structure.

The experimentally observed regularities of interaction can be split into two stages: (1) the shear stress τ depends on the strain u; (2) such a dependence is not observed. In order to describe the first stage, the model of a standard linear body is used as the basis. At the second stage, the law of Coulomb dry friction obviously holds. It should be noted that Winkler-type laws and the Kelvin–Voigt law were also taken as the basis at the first stage. However, the mathematical model of interaction developed on the basis of the latter two laws has some drawbacks of fundamental nature. According to the results of experiments, the return motion of a solid relative to the soil (unloading) obeys a Winkler-type law.

Thus, the following relations are proposed for describing the interaction of solids with soils for $\sigma_N > 0$ (for $\sigma_N \leq 0$, $\tau \equiv 0$):

$$\frac{d\tau}{K_x^D(\sigma_N, I_s)\,dt} + \mu_s(\sigma_N, I_s, \dot{u})\frac{\tau}{K_x^s(\sigma_N, I_s)} = \frac{du}{dt} + \mu_s(\sigma_N, I_s, \dot{u})\,u \tag{2.1}$$

for $0 \leq u \leq u_*$, $du/dt \geq 0$;

$$\tau = f(\dot{u})\sigma_N \tag{2.2}$$

for $u > u_*$, $du/dt \geq 0$;

$$\frac{d\tau}{K_x^R(\sigma_N, I_s)\,dt} = \frac{du}{dt} \tag{2.3}$$

for $u < u_*$, $du/dt < 0$, and

$$\tau \equiv 0. \tag{2.4}$$

for $u \geq u_*$, $du/dt < 0$. Here $K_x{}^D(\sigma_N, I_s)$ is the dynamic coefficient of interaction for $du/dt \to \infty$ and $K_x{}^s(\sigma_N, I_s)$ is the static coefficient for $du/dt \to 0$; $\mu_s(\sigma_N, I_s, \dot{u})$ is the parameter of shear viscosity of the soil, defined as

TABLE 1

Soil	K_N^s, m^{-1}	K_N^D, m^{-1}	β	f	φ
Loess with undistorted structure	100	400	2,5—3,0	0,6	2,5
Loess with distorted structure	100	400	2,0	0,5	2,0
Sand with distorted structure	150	350	2,5—3,0	0,5	1,7
Loam with distorted structure	160	450	2,0	0,7	2,0

$$\mu_s(\sigma_N, I_s, \dot{u}) = \frac{K_x^D(\sigma_N, I_s) K_x^s(\sigma_N, I_s)}{[K_x^D(\sigma_N, I_s) - K_x^s(\sigma_N, I_s)]\eta_s(\sigma_N, I_s, \dot{u})};$$

$\eta_S(\sigma_N, I_S, \dot{u})$ is the shear viscosity; $\dot{u} = du/dt$; $I_S = |u/u_*|$ is the parameter characterizing the distortion of the soil structure, $f(\dot{u})$ is the friction coefficient which is a function of the rate of relative displacement in the general case; and $K_xR(\sigma_N, I_S)$ is the coefficient of interaction for the return motion of a solid relative to the ground.

The values of the coefficients of interaction, which are functions of the normal pressure and the structure of the soil, are defined as

$$K_x^D(\sigma_N, I_s) = K_x^{D*}(\sigma_N)\exp[\beta(1 - I_s)]; \quad (2.5)$$

$$K_x^s(\sigma_N, I_s) = K_x^{s*}(\sigma_N)\exp[\alpha(1 - I_s)]; \quad (2.6)$$

$$K_x^R(\sigma_N, I_s) = K_x^{DN}/(1 - I_s), \quad (2.7)$$

where K_x^{D*} and K_x^{s*} are the shear coefficients of the dynamic and static interaction for $u = u_*$; K_x^{DN} and K_x^{sN} are the initial values of the coefficients of interaction; and β and α are the coefficients characterizing the ranges of K_x^D and K_x^s.

From relations (2.5) and (2.6), we obtain

$$K_x^{DN} = K_x^{D*}\exp(\beta), \quad K_x^{sN} = K_x^{s*}\exp(\alpha). \quad (2.8)$$

Assuming that

$$\gamma_N = K_x^{DN}/K_x^{sN}, \quad \gamma_* = K_x^{D*}/K_x^{s*}, \quad (2.9)$$

from (2.8) we obtain

$$\alpha = \beta + \ln(\gamma_*/\gamma_N). \quad (2.10)$$

Since $\gamma_* > \gamma_N$, we have $\alpha \geq \beta$.

According to the experimental results, the value of the shear coefficient of interaction K_x^{s*} linearly depends on the normal pressure σ_N:

$$K_x^{s*} = K_N^s\sigma_N, \quad K_x^{D*} = K_N^D\sigma_N \quad (2.11)$$

(K_N^D and K_N^s are the dynamic and static coefficients characterizing the rigidity of bond (coupling) between the particles of soil and a solid). The experimental results show that the displacement $u_* = 0.5\cdot 10^{-2}$-$0.7\cdot 10^{-2}$ m and is constant for a given type of soil.

Using the condition $u_* = \text{const}$ and (2.1), we find that

$$\mu_s^* = A/\gamma_*\theta, \quad (2.12)$$

where A is a dimensionless coefficient and θ the characteristic time of action of the shearing load. For the types of soil under investigation, A = 700-1000.

In the course of interaction, the value of γ_* increases since the distortion of the soil structure leads to an increase in the difference between K_x^D and K_x^s and is defined by the formula

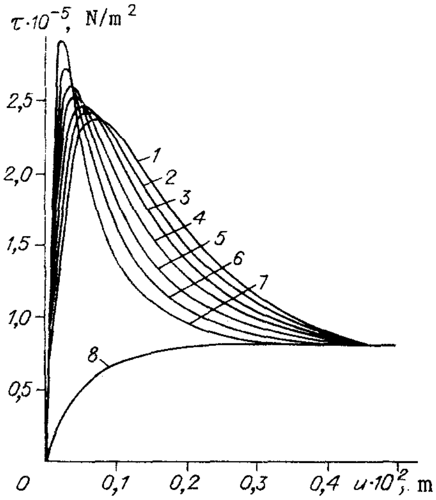

Fig. 6

$$\gamma_* = \gamma_N + (\gamma_*^m - \gamma_N)\left(\frac{\dot{u}}{c_s}\right)^{\varkappa}. \tag{2.13}$$

Here γ_*^m is the maximum possible value of γ_* for $\dot{u} = c_s$, c_s is the velocity of propagation of shear waves in the soil, and κ the coefficient expressing the effect of the strain rate on the structure of the soils. For the soils investigated in our experiments, $\kappa = 0.1$, $\gamma_*^m = 4\text{-}10$.

The shear viscosity parameter decreases upon a change in the soil structure [6]. The value of μ_s can be determined from the relation

$$\mu_s(\sigma_N, I_s, \dot{u}) = \mu_s^* \exp[\varphi(1 - I_s)] \tag{2.14}$$

(φ is the exponent of the change in the value of shear viscosity of the soil).

The $\tau_r(\sigma_N)$ dependences plotted according to the experimental results indicate that they satisfy the following relation proposed in [18]:

$$\tau_r = \tau_{r0} + f\sigma_N/(1 + f\sigma_N/(\tau_{r*} - \tau_{r0})) \tag{2.15}$$

(τ_{r0} is the force of adhesion between the solid and the soil and τ_{r*} the limiting value of the frictional force). The results of experiments proved that τ_{r*} = 0.7-0.9 MPa for $\sigma_N \geq$ 1 MPa. Instead of (2.2), relation (2.15) should be used in the model of interaction for the types of soil under investigation.

The values of the main parameters of the model determined experimentally are compiled in Table 1.

Figure 6 shows the $\tau(u)$ dependences obtained from the solution of Eqs. (2.1)-(2.4). By specifying the law of variation of displacement in the form $u = vt$ ($v = \dot{u}$ = const, t is the time) and using also relations (2.5)-(2.15), we calculated the value of the shear stress from Eqs. (2.1)-(2.4). Curves 1-7 correspond to the strain rate v = 0.0001; 0.001; 0.1; 1; 10; and 100 m/sec; here, $\beta = 3.0$, $\varphi = 0$, $f = 0.5$, $K_N^s = 100$ m^{-1}, $\sigma_N = 0.15$ MPa, $\gamma_N = 1.1$, $\gamma_*^m = 10$, $A = 1000$, $c_s = 100$ m/sec, $\kappa = 0.1$. It can be seen that the values of τ_p and the intensity of distortion of the soil structure increase with v. Taking into account the variation of the shear viscosity with the structure of the soil according to (2.14) (for $\varphi = 2$), we find that an increase in the strain rate does not affect the value of τ_p. The peak strength of the soil is not manifested during interaction of a solid with a soil having a distorted structure ($\beta = 2$ and $\gamma_N = 3$). In this case, the change in v does not affect the $\tau(u)$ dependence (curve 8).

An analysis of Eqs. (2.1)-(2.4) for various values of the model parameters, normal stress σ_N and the laws of variation of $u = \psi(t)$ indicates that relations (2.1)-(2.4) successfully describe the interaction of solids with soils.

Thus, the results of experimental investigations of shear interaction between solids and the soil lead to a mathematical model taking into account the main features of the interaction process observed in experiments.

LITERATURE CITED

1. Tashkent Earthquake of April 26, 1966 [in Russian], Fan, Tashkent (1971).
2. K. S. Sultanov and T. É. Yusupov, Izv. Akad. Nauk UzSSR, Ser. Tekhn. Nauk, No. 3 (1986).
3. D. D. Barkan, Dynamics of Foundations [in Russian], Stroivoenmorizdat, Moscow (1948).
4. T. R. Rashidov, Dynamic Theory of Earthquake Resistance of Complex Systems of Underground Constructions [in Russian], Fan, Tashkent (1973).
5. K. S. Sultanov and V. Yu. Kim, FTPRPI, No. 1 (1986).
6. S. S. Vyalov, Rheological Foundations of Soil Mechanics [in Russian], Vyssh. Shkola, Moscow (1978).
7. A. W. Bishop, in: Fundamental Laws in Soil Mechanics [Russian translation], Mir, Moscow (1975).
8. N. A. Tsytovich, Soil Mechanics [in Russian], Vyssh. Shkola, Moscow (1983).
9. N. R. Barton, Int. J. Rock Mech. Mining Sci. Geomech. Abstr., 13, No. 9 (1976).
10. S. C. Bandis, A. C. Lumsden, and N. R. Barton, Int. J. Rock Mech. Mining Sci. Geomech. Abstr., 20, No. 2 (1983).
11. S. S. Grigoryan, K. A. Gulakyan, A. V. Savinkov, and M. V. Melkumyan, in: Problems of Soil Thermal Mechanics [in Russian], Izd. MGU, Moscow (1986).
12. N. A. Tsytovich, A. L. Kryzhanovskii, T. Rakhmanov, and Sh. Zh. Surankulov, Inzh. Geol., No. 6 (1983).
13. V. S. Nikiforovskii and E. I. Shemyakin, Dynamic Fracture of Solids [in Russian], Nauka, Novosibirsk (1979).
14. V. N. Nikolaevskii, in: Fundamental Laws of Soil Mechanics [Russian translation], Mir, Moscow (1975).
15. S. S. Grigoryan, Dokl. Akad. Nauk SSSR, 244, No. 4 (1979).
16. M. G. Tseitlin and A. A. Kosheleva, Osnovaniya, Fundamenty Mekhanika Gruntov, No. 3 (1984).
17. R. I. Nigmatulin and N. N. Kholin, Izv. Akad. Nauk SSSR, MTT, No. 4 (1974).
18. A. A. Vovk, B. V. Zamyshlyaev, L. S. Evterev, et al., Behavior of Soils under Pulsed Loads [in Russian], Naukova Dumka, Kiev (1984).